HIGHER MATHEMATICS - 2013

Time : 3 Hours] **Class : 12th** [M. M. : 100

Instructions-

(1) **All questions are compulsory.**

(2) **Read the instruction of question paper carefully and write answers of them.**

(3) **There are two Sections-Section A and Section B in the question paper.**

(4) **In Section A Question Nos. 1 to 5 are objective type which contain Fill in the blanks, True/False, Match the columns and Choose the correct answers. Each question is allotted 5 marks.**

(5) **Internal options are given in Question Nos. 6 to 21 of Section B.**

(6) **Q. Nos. 6 to 12 carry 4 marks each.**

(7) **Q. Nos. 13 to 19 carry 5 marks each.**

(8) **Q. Nos. 20 and 21 carry 6 marks each.**

Section-A

Q. 1. Choose the correct answers from the given options of objective type questions:

(i) Partial fractions of $\frac{1}{x(x+a)}$ are:

(a) $\left[\frac{1}{x}-\frac{1}{x+a}\right]$ (b) $\frac{1}{a}\left[\frac{1}{x}+\frac{1}{x+a}\right]$

(c) $\frac{1}{a}\left[\frac{1}{x}-\frac{1}{x+a}\right]$ (d) $\left[\frac{1}{x}+\frac{1}{x+a}\right]$

(ii) Value of $\cos^{-1}\frac{3}{5}-\sin^{-1}\frac{4}{5}$ is:

(a) 1 (b) 0

(c) -1 (d) $\frac{4}{5}$

(iii) The perpendicular distance of the point (5, 12, 13) from x axis is:

(a) $\sqrt{313}$ (b) $\sqrt{13}$

(c) $\sqrt{5}$ (d) $\sqrt{12}$

(iv) Find the equations of the plan which is parallel to X axis and cuts the intercept 5 and 7 respectively on Y and Z axis:

(a) $5x+3y=15$ (b) $7x+3z=21$

(c) $7y+5z=35$ (d) $35x+21y+15z=105$

(v) Find the equation of straight line passing through the points (2, 3, 4) and (1, -2, 3):

(a) $\frac{x-2}{1}=\frac{y-3}{-5}=\frac{z-4}{-1}$ (b) $\frac{x-2}{-1}=\frac{y-3}{-5}=\frac{z-4}{-1}$

(c) $\frac{x-2}{-1}=\frac{y-3}{5}=\frac{z-4}{-1}$ (d) $\frac{x-2}{-1}=\frac{y-3}{-5}=\frac{z-4}{1}$

Q. 2. Choose True/False in the following statements:

(i) The co-ordinates of the ends of a diameter of a sphere are (2, -3, 4) and (-5, 6, 7) and its equation is $x^2+y^2+z^2+3x-3y+3z-56=0$.

(ii) The starting and end points of a vector are collinear, it is known as unit vector.

(iii) When a body is thrown up, the sign of g is negative when it goes up.

(iv) The coefficient of correlation between two variables x and y is:

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$$

(v) The arithmetic mean of regression coefficients is less than coefficient of correlation.

Q.3. Give the answer in one word each:

(i) Displacement is which quantity in vectors algebra?

(ii) What is the differential coefficient of log (log sin x)?

(iii) If the limits of an integral are mutually changed, then what is the effect of sign on integral?

(iv) What type of quantity work is?

(v) Write the formula of square root of a quantity y by Newton-Raphson method.

Q.4. Match the columns:

Column 'A'	**Column 'B'**
(i) The relation between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$	(a) $\frac{45}{16}$
(ii) $\int \sin^2 x.dx$	(b) $1 - \frac{\pi}{4}$
(iii) $\int_0^{\pi/4} \tan^2 x.dx$	(c) $\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)$
(iv) If x_n is a nearer root of equation $f(x) = 0$, then by Newton-Raphson method x_{n+1} is	(d) Parallel
(v) The root of the equation $x^2 - x - 5$ near to $\frac{5}{2}$ by Newton-Raphson method is	(e) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Q.5. Fill in the blanks:

(i) Differential coefficient of $\sin x^3$ will be.....

(ii) The value of $\int \frac{1}{1-\sin x} dx$ is .....

(iii) Area of circle $x^2 + y^2 = a^2$ is.....

(iv) Method of imaging position is.....

(v) After first repetition the cube root of 10 by Newton-Raphson method is.....

Section-B

Q.6. Break $\frac{x^4}{x^3+1}$ in to partial fractions. **(Or)**

Break $\frac{2x+1}{(x-1)(x^2+1)}$ in to partial fractions.

Q.7. If $\sin^{-1} x + \tan^{-1} x = \frac{\pi}{2}$ then prove that:

$$2x^2 = \sqrt{5} - 1$$ **(Or)**

Prove that:

$$\cos^{-1}\frac{3}{11} - \sin^{-1}\frac{3}{4} = \sin^{-1}\frac{19}{44}$$

Q.8. Find the differential coefficient of $\tan^{-1} x$ by first principle. **(Or)**

Differentiate w.r. to x, the following function:

$$\log\sqrt{\frac{1+\cos x}{1-\cos x}}$$

Q.9. Verify Rolle's theorem for the function $f(x) = \sin x + \cos x - 1$ in the interval $\left[0, \frac{\pi}{2}\right]$. **(Or)**

Prove that the maximum value of $\sin x + \cos x$ is $\sqrt{2}$.

Q.10. Prove that coefficient of correlation lies between - 1 and 1. **(Or)**

Find the coefficient of correlation by step deviation method from the given data:

x	y
10	18
14	12
18	24
22	6
26	30
30	36

Q. 11. The data are received from a firm for advertisement and sale as given below:

	Sales (in crores)	Advertisement (in crores)
Mean	40	6
Standared deviation	10	1.5

Coefficient of correlation r = .9

If the firm fixed the target for sale as 60 crores rupees, then calculate the amount for advertisement. **(Or)**

Find the line of regression of y on x for the following data:

x	y
10	8
9	12
8	7
7	10
6	8
4	9
3	6

Q. 12. If $x^y = e^{x-y}$, then prove that:

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$ **(Or)**

If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)\frac{d^2y}{dx^2} - x.\frac{dy}{dx} - 2 = 0.$

Q. 13. Find the equation of planes which passes through the intersection of the lines x + 3y + 6 = 0 and 3x - y - 4z = 0 and their perpendicular distance from the origin is 1. **(Or)**

A variable plane keeping at a constant distance P from the origin O cuts the intercept on the axis at the points A, B. C. Show that the locus of centroid of tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2}$ $16P^{-2}$

Q. 14. Prove by the vector method, the law of sine in trigonometry:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

(Or)

Show that:

$$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$$

Q. 15. Find the value of:

$$\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$$

(Or)

$f(x) = \dfrac{\sin 2x}{x}$ when $x \neq 0$. $f(x) = 1$ when $x = 0$, whether $f(x)$ have continuity at $x = 0$

Q. 16. Find the value of:

$$\int \frac{5x + 7}{\sqrt{3 - 2x - x^2}} dx$$

(Or)

Find the value of:

$$\int \frac{dx}{1 - 2\sin x}$$

Q. 17. Prove that:

$$\int_0^{\pi/2} \frac{1}{1 + \tan^3 x} dx = \frac{\pi}{4}$$

(Or)

Find the area between curve $x^2 = 4y$ and the line $x = 4y - 2$.

Q. 18. Solve the differential equation:

$x(x - y)dy + y^2 dx = 0$.

(Or)

Solve the differential equation:

$$\cos x \frac{dy}{dx} + y = \sin x$$

Q. 19. In a group of 10 children, there are 6 boys and 4 girls, 3 children are selected at random. Find the probalitiy that the selected group have only one special girl.

(Or)

Two cubical dice are thrown simultaneously. Find the probability of getting an even number of first dice or getting the sum 9 on two dice.

Q. 20. Show that the lines:

$$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7} \text{ and } \frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$$

interesect each other. Find the co-ordinates of intersecting point also. **(Or)**

Find the minimum distance between the lines:

$$\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1} \text{ and } \frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}$$

Q. 21. Find the equation of plane passing through the point (1, 2, 3) and through the intersecting line of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})+5=0$ and $\vec{r}.(2\hat{i}-4\hat{j}+\hat{k})=3$ **(Or)**

Find the minimum distance between the lines whose vector equations are:

$\vec{r}=\hat{i}+\hat{j}+\lambda(2i-\hat{j}+\hat{k})$ and $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$.

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